ARIEL UNIVERSITY

אוניברסיטת אריאל בשומרון

# Foundations of Modern Cryptography

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Neapolis University Pafos

Lecture 6: November 6, 2024

# Lecture Plan

- Digital signatures.

If the screen seems frozen and I do not respond,
please call me in Telegram.

# Task: Digital Signatures

Alice

public key, secret key

Bob

—

Charlie

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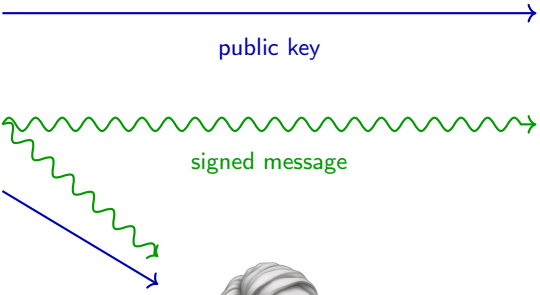


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Charlie should be unable to sign (not to say recover the secret key) even though he has seen a lot of signed messages.

Pictures courtesy AI Emojis Generator

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Perhaps his partner even forced Alice to sign similar messages!

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## Definition (Digital signatures scheme, DSS)

. . . consists of polynomial-time algorithms $G, S, V$:

- $G$: $(1^n, r_g) \mapsto (s, v)$ (secret signing key, public verification key),
- $S$: $(s, \texttt{msg}) \mapsto \texttt{sign}$ (signing algorithm),
- $V$: $(v, \texttt{msg}, \texttt{sign}) \mapsto$ yes or no,

Correctness:

- $\forall\ \texttt{msg}\ \mathbf{Pr}_{r_g}\{V_v(\texttt{msg}, S_s(\texttt{msg})) = 0\} = 0.$

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- $\ell(n)$-bounded scheme: limited to messages of length $\ell(n)$.
- General schemes.

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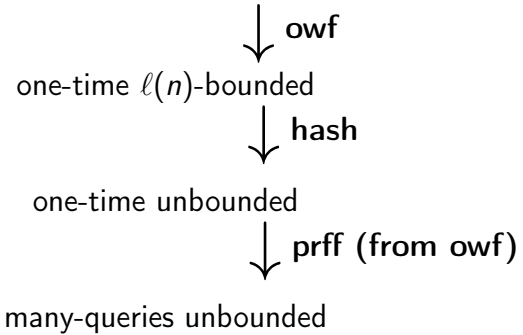
## Exercise

Does it matter whether $S, V$ are deterministic or randomized?

# Workflow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## Theorem

Let $f$ be a owf, $m = \ell(n)$ is a polynomial. Let $s_k^i \leftarrow U_n$, compute $v_k^i = f(s_k^i)$,

$$G(1^n) = (\underbrace{(s_1^0, s_1^1, \ldots, s_m^0, s_m^1)}_{\mathtt{sk}}, \underbrace{(v_1^0, v_1^1, \ldots, v_m^0, v_m^1)}_{\mathtt{pk}}).$$

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Let

$$S_s(\underbrace{\beta_1 \ldots \beta_m}_{\mathtt{msg}}) = (s_1^{\beta_1}, \ldots, s_m^{\beta_m}).$$

Let $V(\mathtt{msg}, \mathtt{sign})$ check that

$$\forall i \quad f(\mathtt{sign}_i) = v_i^{\beta_i}.$$

Then $(G, S, V)$ is a secure one-time $\ell(n)$-DSS.

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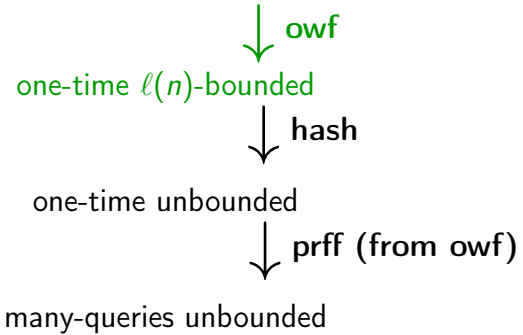
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## Definition (Collision-resistant hash-function family, crhff)

$\ell(m)$-size crhff $\{h_\xi\}_\xi$ is given by polynomial-time $Z, H$ such that

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## Exercise

1. Give an equivalent definition without $H$.
2. Show that crhff can be constructed from owf.

# One-time **un**bounded scheme: Construction

## Theorem

Assume that $(G, S, V)$ is a secure one-time $\ell(m)$-bounded DSS and $\{h_\xi\}_\xi$ is a $\ell(m)$-crhff generated by $Z$. Then the following construction provides a secure one-time unbounded DSS:

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## Exercise

Prove it (hint: we must break *either* $(G, S, V)$ or $\{h_\xi\}_\xi$).

# One-time **un**bounded scheme: Idea, version 2

## Hash-and-sign paradigm, revisited; one-way hash-function family

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- $H(\xi, x) = h_\xi(x)$
- $\forall$ adversaries $M, A$

$$\mathbf{Pr}_{y \leftarrow M(1^n),\ \xi \leftarrow Z(1^n)}\{A(\xi) = y' : h_\xi(y) = h_\xi(y'),\ y \neq y'\} < \varepsilon(n)$$

- $y$ is chosen before $\xi$.

## Exercise

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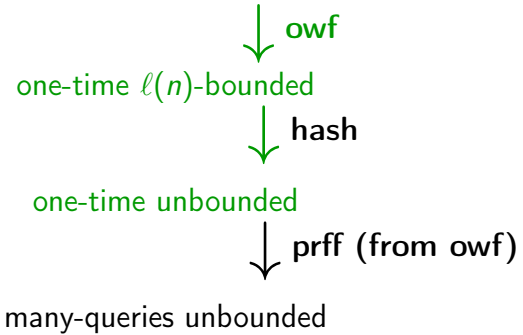
3. Prove that it is enough for securing the DSS construction.

# Workflow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- No memory $\Rightarrow$ will regenerate key like $v_{0100010}$.
- To avoid signing two messages by the same key use prff instead of a truly random path. (Otherwise add this negligible probability of forgery.)
- We will still be using prff instead of the true randomness for $G$ and $S$ to be able to regenerate keys (and their certificates) without memory.

# Many-time DSS: Construction

## From one-time DSS and pseudorandom function family

- Use one-time $(G, S, V)$ (w.l.o.g., $G$ and $S$ require at most $n + 3$ random bits) and prff $Z$:
  - $(s, v) \leftarrow G$, "root key"
  - $\zeta \leftarrow Z$

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- Signing key $\mathtt{sk}$: $(\zeta, s)$
- Verification key $\mathtt{pk}$: $v$
- Signing algorithm
  - Select path $\sigma = \sigma_1 \dots \sigma_n \in \{0,1\}^n$ from the root key to a leaf either at random or using $\zeta(110\xi(\mathtt{msg}))$.
  - Generate all the keys (both left and right) for prefixes of this path ($i = 0, 1, \dots, n-1$):

$$(s_{\sigma'0}, v_{\sigma'0}) \leftarrow G(1^n, \zeta(10^{n-i+1}\sigma'0)) \text{ for } \sigma' = \sigma_1 \dots \sigma_i,$$
$$(s_{\sigma'1}, v_{\sigma'1}) \leftarrow G(1^n, \zeta(10^{n-i+1}\sigma'1)) \text{ for } \sigma' = \sigma_1 \dots \sigma_i,$$

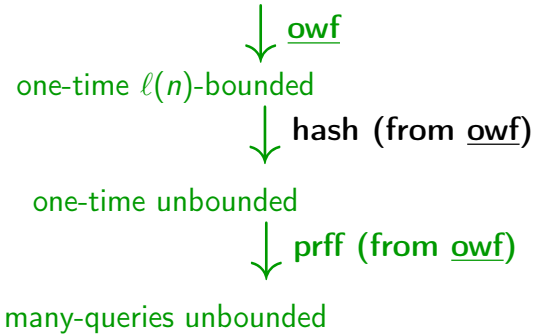
  - Given keys $(s_\sigma, v_\sigma)$:

$$S_{s_\sigma}(\mathtt{msg}, \underbrace{\zeta(00^{\cdots}\sigma)}_{\text{randomness for } S}).$$

  - Keys are signed sequentially:

$$\begin{array}{llll} (v_0, & v_1, & S_s(v_0 & \circ v_1 \quad , \zeta(00^{n+2}))), \\ (v_{\sigma_1 0}, & v_{\sigma_1 1}, & S_{s_{\sigma_1}}(v_{\sigma_1 0} & \circ v_{\sigma_1 1} \quad , \zeta(00^{n+1}\sigma_1))), \\ & \dots & & \\ (v_{\sigma_1 \dots \sigma_{n-1} 0}, & v_{\sigma_1 \dots \sigma_{n-1} 1}, & S_{s_{\sigma_1 \dots \sigma_{n-1}}}(v_{\sigma_1 \dots \sigma_{n-1} 0} & \circ v_{\sigma_1 \dots \sigma_{n-1} 1} \quad , \zeta(00^3\sigma_1 \dots \sigma_{n-1}))). \end{array}$$

# Workflow


owf
one-time $\ell(n)$-bounded
hash (from owf)
one-time unbounded
prff (from owf)
many-queries unbounded

# Reminder: Is Cryptography Possible?

## Impagliazzo's Worlds

Russell Impagliazzo

(taken at CSR-2006, SPb, by A. Kulikov)

Algorithmica

**P** = **NP**, no crypto

**P** ≠ **NP**, how to use?

Heuristica

Easy on the average

Trapdoor functions

Cryptomania

public-key encryption

digital money

Hard on the average

. . . also hard to generate

Pessiland

No one-way functions

One-way functions

Minicrypt

digital signatures

private-key crypto

# Takeout

- One-time length-bounded digital signatures from owf.
- One-time unbounded digital signatures, two constructions:
  - collision-resistant hash-function family,
  - one-way hash-function family.
- Multiple-query unbounded digital signatures (using prff).

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- One-time length-bounded digital signatures from owf.
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  - one-way hash-function family.
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*Coming next:*

- *DEADLINE (homework).*
- *Midterm.*