ARIEL UNIVERSITY

אוניברסיטת אריאל בשומרון

Foundations of Modern Cryptography

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Lecture 9: December 12, 2024

Lecture Plan

- How ZK helps to force malicious participants to be semi-honest.
- (maybe) Clarifications regarding cryptographic primitives.

If the screen seems frozen and I do not respond,
please call me in Telegram.

Task: Zero-Knowledge Proof

Alice

x, proof that $x \in L$

Bob

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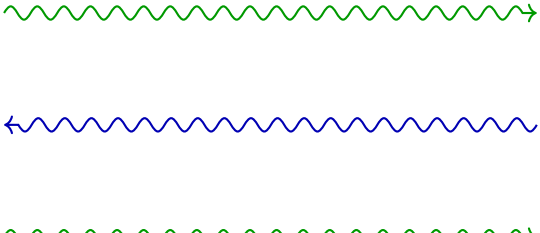

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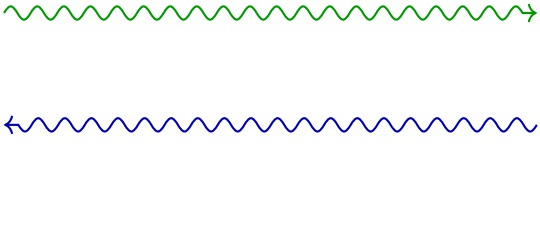

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Pictures courtesy AI Emojis Generator

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Bob gets convinced iff $x \in L$ (with certain probability of error).
Bob does not learn from this protocol about Alice's proof more than the fact $x \in L$.

Strong ZK: Bob also gets convinced that Alice indeed knows some proof that Bob could verify.

In the previous lecture we have seen such proofs for **NP** languages.

Pictures courtesy AI Emojis Generator

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- Delegate sampling according to a polynomial-time samplable distribution.
- Input commitment with provably random keys and with optional opening.
- Show it's enough for Yao's secure 2-party computation protocol.

Forcing uniform randomness: tossing a coin together

Failed attempt:

- Alice selects $a \in U_1$, Bob selects $b \in U_1$.
- They exchange the bits.
- Now $a \oplus b$ is distributed as U_1.

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Correct scheme:

- Alice selects $a \in U_1$, Bob selects $b \in U_1$.
- Alice sends a commitment $C_s(a)$ for a keeping the key s in secret.
- Bob sends b.
- Alice sends s.
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Exercise

Prove formally that $a \oplus b$ is distributed almost as U_1. In particular, the so-generated strings are computationally indistinguishable from U_n.

Delegating the computation, simple version

TASK: delegate computing $f(x)$ without knowing x; public checksum

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SCHEME:

- $L = \{(u, v) \mid \exists x \; u = f(x), \; v = h(x)\} \in \mathbf{NP}$.

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SCHEME:

- $L = \{(u, v) \mid \exists x \; u = f(x), \; v = h(x)\} \in \mathbf{NP}$.
- Alice knows (u, v) because she knows x and $f, h \in \mathbf{P}$.
- Bob knows (u, v) because he receives it.
- Thus we can use the CZK protocol from the previous lecture.

Exercise

Prove it formally ...

Hit the Image: Provably send the value without revealing the preimage

TASK: send a value in $\mathrm{Im}\, f$ without revealing the preimage

- Alice generates the value of $f(x)$ and sends it to Bob.
- Bob gets convinced he received a value in $\mathrm{Im}\, f$ while learning nothing about x.

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SCHEME:

- $L = \operatorname{Im} f$.
- They both know f.
- Thus we can use the Strong CZK protocol from the previous lecture.
- Bonus: Bob is also convinced that Alice knows (at least one) x.

Exercise

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TASK: delegate computing $f(x)$ without knowing x, private checksum

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- If $h(x) \neq y$, Bob still does not learn anything about x beyond $f(x), h(x)$.

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- Alice sends $f(x), h(x)$ as an image point of (f, h).

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- If Bob did not know $h(x)$, this fact does not help him to reveal anything but $h(x)$.

Exercise

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- Alice chooses $t_1, \dots, t_\ell \in U_n$ and $r' \in U_\ell$,
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- Alice and Bob jointly generate $r'' \leftarrow U_\ell$.
- Alice computes $r := r' \oplus r''$.
- Bob delegates the computation of $s(r)$ to Alice and gets the value:
  $f(r', t, r'') = s(r' \oplus r'') \longleftarrow$ Alice computes,
  $h(r', t, r'') = (C_t(r'), r'') \longleftarrow$ Bob already knows.

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- A and B generate Alice-known r, t and B gets $C_t(r)$ *// polynomial-time sampling*
- A sends $C_r(x)$ while Bob holds A's commitments for [some] r and x and can check they have been used in the computation *// delegated computing $C_r(x)$ matching all the three*
  - Bob learns nothing about x or r
  - final commitments to parts of x can be verified upon request
  - their secret keys are random (guaranteed by the sampling protocol: they are the seeds)

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Exercise

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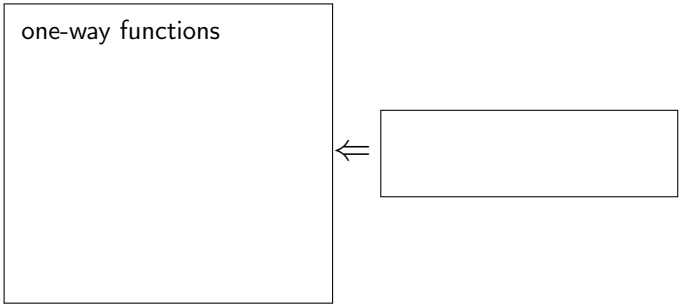
Clarifications regarding cryptographic primitives



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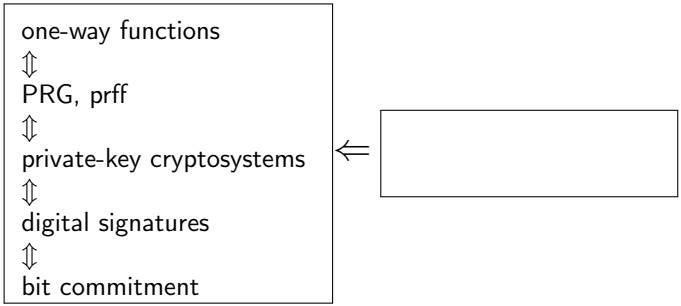

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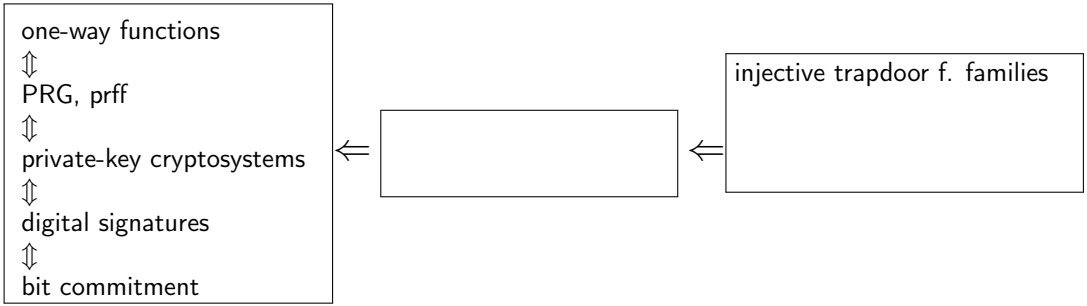

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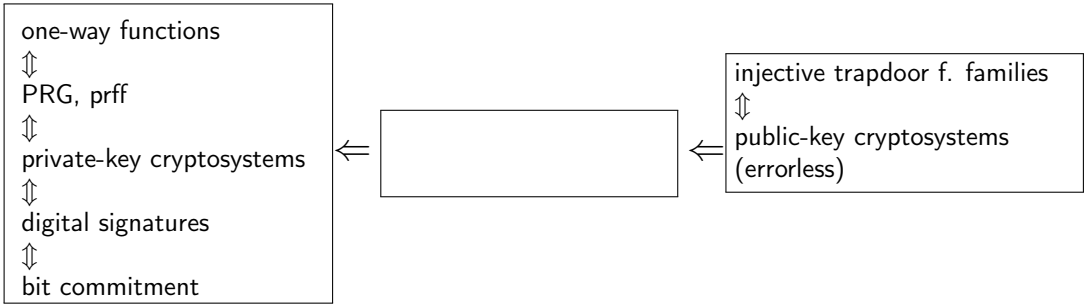

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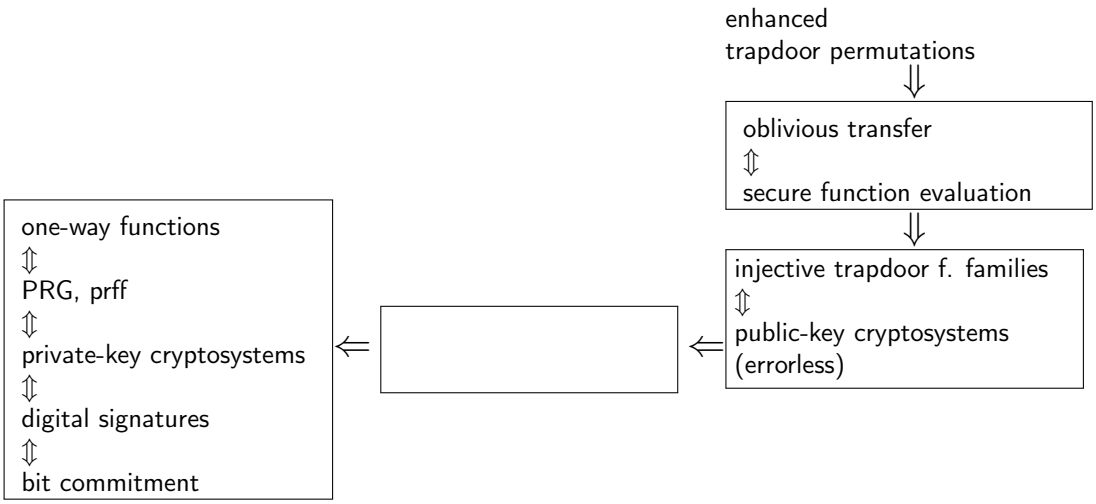

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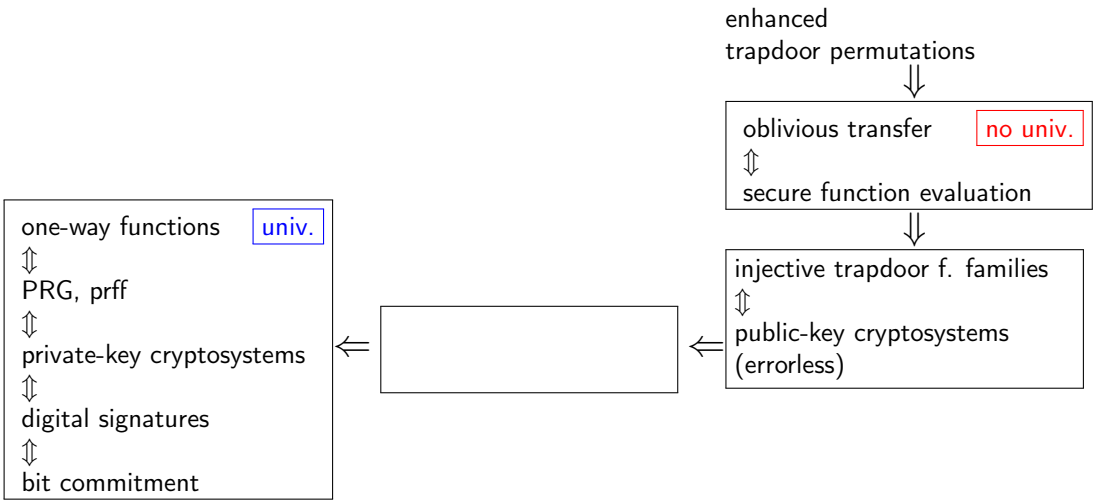
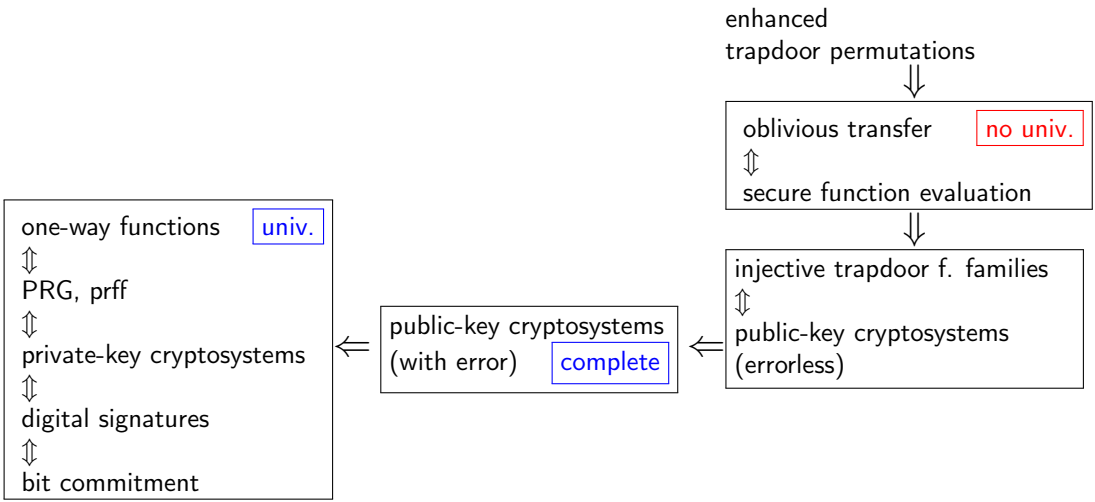


Clarifications regarding cryptographic primitives



Takeout

- Secure two-party computation with malicious adversaries.
- In particular, how to delegate computations with partially unknown data.
- Cryptographic primitives: what implies what.

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Coming next:

- *(maybe) Exercises and Repeat.*
- *Blockchains.*
- *(maybe) The real grounds of the security in modern cryptography.*
- *(maybe) A complete (universal) public-key cryptosystem.*